Characterization of a 2D Face-Recognition Method based on Landmarks Position

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Abstract
This work is focused on the theoretical–statistical characterization of an authentication procedure for human faces that uses few face–landmarks coordinates anyway extracted from 2D face images with neutral facial–expression. The measurement uncertainty of landmarks position is due to noise sources present both in the acquisition system and in the features extraction process. This uncertainty affects the reliability of the recognition method that is expressed in terms of probability of true recog-nition (*PTR*) and false recognition (*PFR*). The authentication problem is approached by using a threshold method based on the Likelihood Ratio Test (LRT). It is an optimal detection technique, as according to the Neymann–Pearson (NP) theorem, that guarantees the minimum achievable *PFR* for a target *PTR* independently of the algorithm used for extracting features.
In particular, this paper provides a theoretical criterion for determining the threshold value that *a–priori* guarantees the desired *PTR* from the knowledge of both measurements uncertainty and number of the used landmarks. Moreover, a given *PFR* value is assured on the basis of the target likeness degree to be discriminated between the probe and the gallery landmarks. Theoretical results are validated by means of Monte–Carlo simulations and are effectively applied also to experimental data of the Bosphorous database.
Keywords :2–D biometrics, Face recognition, face authentication, facial landmarking, likelihood ratio Test (LRT).

I. Introduction

Automatic human face recognition through two-dimensional (2D) images can refer to different application scenarios. One of them is the recognition of a face in a picture that shows many different shapes, another one aims to recognize if the face of a person is stored in a given database [1], [2], [3]. In this paper, the second scenario is considered. In particular, the "authentication" problem is addresses by using the position of few landmarks, extracted from 2D–images of faces, for authenticating the probe–face under test, t, against a gallery–face with claimed identity, j [2], [4], [5], [6]. Within the face authentication context, statistical perfor-mance are typically expressed by the probability of true recognition (P T R) and false recognition (P F R) [5], i.e. the probability of authenticating t with j correctly or wrongly, respectively. Both wrong, false and missed authentication outcomes are due to the random nature of available measured landmarks, which are subjected to the uncertainty sources introduced by both the used acquisition system and the extraction algorithms. In fact, many parameters of the image acquisition, such as illumination, change of the subject position, face–occlusion and face–expression, influence measurements values of the used features and their corresponding uncertainty value [7] and introduce a noisy component independent of the used feature–extraction process. On the other hand, the accuracy of the whole recognition technique is affected also by the feature extraction algorithm selected for processing the acquired face images. In fact, when different extraction algorithms are applied to the same 2D–face image, they could determine different values of the same measured feature [6],[1].

It should be noticed that some source of variability, such as the expression or the position of the probe subject, can not be controlled or compensated for automatically and thus measurements uncertainty, and consequently also the relia-bility of the authentication process where measurements are used, can not be arbitrarily improved. As a consequence, the theoretical characterization of a recognition method results to be a difficult issue since it depends strictly on the uncertainty value of available measurements which can not be reduced or controlled a–priori, neither if the feature extraction process is known. In order to cope with such an issue, scientific literature reports many different benchmarks of landmarking accordingly to the used database [1], [4], [5], [6].

A theoretical characterization of a recognition system that uses feature–angles vector is presented in [5] but recognition performance, although presented in terms of PTR and PFR, are not related to uncertainty of available measurements. On the other hand, [4] evaluates influence of landmarks variance on recognition performance and proposes a new selection criterion for choosing landmarks to be used in the face–classification process. In particular, [4] proves that features affected by larger noise variance do not significantly improve accuracy of the recognition technique. The recognition accuracy evaluated in [4] is given in terms of PTR, which has been estimated by using experimental results, while corresponding PFR is not analyzed.

In this context, this paper characterizes the statistical properties of a face–authentication algorithm that employs Cartesian coordinates of few landmarks, anyway extracted, discriminating a human face which is supposed to assume a neutral expression. Evaluated performance concern only the authentication procedure and is expressed in terms of both P T R and PFR. In particular, the PTR is related to the measurement variance, and consolidates experimental results shown in [4]. Theoretical P F R is also given and related to the degree of likeness between the two subjects and the number of used landmarks. Since the characterization of the method is

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performed independently on the landmarking technique, thus the analyzed authentication algorithm, and its corresponding theoretical characterization, are an effective tool for comparing performance of different features extraction algorithms.

The recognition process used in this work follows the Likelihood Ratio Test (LRT) threshold criterion, which is an optimal detection technique as according to the Neymann–Pearson (NP) theorem [8]. Thus, it is widely used for revealing faults or recognizing events with high reliability level [9], [10], [11]. The LRT design procedure leads to an authentication algorithm already used in the scientific literature [1]–[2]: the claimed identity is authenticated if the algorithm outcome is lower than a given value, γ, that guarantees the quality of correspondence between subjects. Although the algorithm is widely applied, however the criterion for setting the authenticating threshold value is set empirically on the basis of available data measurements [1]–[2]. In this context, this paper uses the LRT theory in order to provide a criterion for setting a–priori the γ value to be used in the authentication procedure on the basis of the desired test PTR, of the knowledge of both the measurements uncertainty, σ^2, and of the number of the used landmarks, N_P. In particular, LRT methodology is explicitly customized for the face–authentication issue that uses position of few landmarks, and test results are opportunely interpreted in terms of target PTR and PFR.

This paper is organized as follows: at first, statical model of the measured landmarks (sec.II) is described. The hypothesis testing that describes the authentication problem is formally defined in sec.III. Moreover, the LRT–based authentication algorithm is designed and the corresponding statistical characterization, in terms of PTR and PFR, is given in closed form (sec.IV). Theoretical results are validated by means of Monte–Carlo simulations and are used for designing the authentication procedure which guarantees a desired PTR value. Application examples to experimental data of the Bosphorous database [12] are presented in sec.V. Conclusions follow.

II. Data model

In this section the problem of *authenticating* a subject from few N_P landmarks coordinates extracted from 2D–images of faces with neutral expression is introduced and the statical model, needed for designing the *authentication* procedure, is given. Data employed for introducing the issue are those provided by the Bosphorous database [12] which are used in this paper with the scope of providing experimental results validating the theoretical analysis and application examples. The use of this particular database is not constraining for the proposed recognition test which instead can be applied to any set of face–landmarks that uses any number of landmarks coordinates indicated by N_P.

Coherently with scope of the paper, in this work only 2D human faces with neutral expressions (i.e. 299 faces) have been considered for which both face–images and face–landmarks, extracted from the corresponding images, are available. In particular, we have taken into account the 22 landmarks that

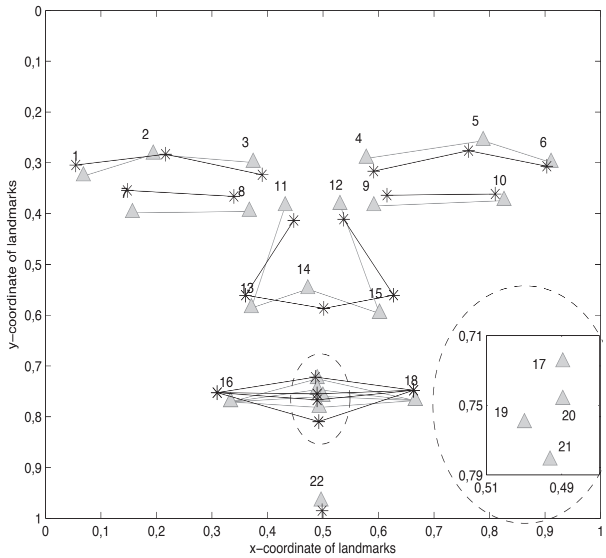

Fig. 1. Normalized face–landmarks of a probe subject t (gray triangles) and of a gallery subject j (black stars) with neutral expression. Data have been extracted by the Bosphorous database. Probe landmarks are of the person labeled as bs077_N_N_0. Gallery landmarks are of the person labeled as bs055_N_N_0.

are present in all subjects and the corresponding coordinate values have been normalized in the range [0, ..., 1]. Moreover, all data have been aligned to the subject bs050_N_N_0 by using the Procrustes transform [13], [14], which is a linear transformation, based on minimum Least square approach, for guaranteeing a fast and precise alignment between homologous faces points. Fig.1 shows an example of normalized face–landmarks of two people with neutral expressions in the Bosphorous database. In particular, landmarks coordinates of the probe subject t (gray triangles), labeled as bs077_N_N_0, have been superimposed on landmarks of the gallery subject j (black stars) of the subject bs055_N_N_0.

In this paper, the i–th landmark of the probe–subject t is identified by its Cartesian coordinates, i.e. $P_{i_t} \triangleq \{x_{i_t}, y_{i_t}\}$, and are modeled as Normal random variables with known variance σ^2 as described by:

$$x_{i_t} \sim \mathcal{N}\left(\mu_{x_{i_t}}, \sigma^2\right), \qquad i = 1, ..., N_P \tag{1}$$

$$y_{i_t} \sim \mathcal{N}\left(\mu_{y_{i_t}}, \sigma^2\right). \qquad i = 1, ..., N_P \tag{2}$$

Every expected value, $\mu_{x_{i_t}}$ and $\mu_{y_{i_t}}$, is an unknown parameter and represents the true value of the corresponding landmark coordinate. All measured coordinates are supposed to be subjected to the same variance value that relies on the algorithm used for extracting features, on the quality of the used image, on technical specifications of camera used for acquiring images, on the environmental lighting condition.

The true value of measurements variance has been set by using an Euclidean criterion on available normalized data, specifically by considering the value 3σ equal to one third of the minimum Euclidean distance, d, between adjacent landmarks, thus obtaining $\sigma^2 = d^2/81$. The variance value obtained for the normalized Bosphorous measurements used in this paper is $\sigma^2 = 2 \cdot 10^{-4}$.

Each coordinate of the gallery–face j available in the database, is supposed to be affected by a measurement uncertainty value which is negligible with respect to σ^2 and thus coordinates of the gallery database can be modeled as deterministic parameters and are indicated as $P_{i_j} \triangleq \{\mu_{x_{i_j}}, \mu_{y_{i_j}}\}$.

The authentication procedure determines if a person under test, t, belongs to the database. Thus, face of t is subjected to the image acquisition process and the corresponding features, composed by the $(2 \cdot N_P)$ measured coordinates modeled by (1)–(2), are extracted. Vectorial symbols are introduced in order to simplify mathematical notation of next sections. In particular, the 2D–face mask of the probe–face t is indicated $\mathbf{M}_t \triangleq \{P_{1_t}, P_{2_t}, ..., P_{N_{P}t}\}$. It follows that the joint probability density function (pdf) of available measurements of t is:

$$p_{\mathbf{M_t}}(\mathbf{M_t}) = \frac{1}{(2\pi\sigma^2)^{2 \cdot N_P}} \cdot \exp\left(-\frac{1}{2\sigma^2}\sum_{i=1}^{N_P}\left(\left(x_{i_t} - \mu_{x_{i_t}}\right)^2 + \left(y_{i_t} - \mu_{y_{i_t}}\right)^2\right)\right) \quad (3)$$

By using this statistical model, the binary hypothesis test proposed for face authentication is described in the next section.

III. Hypothesis testing design for 2D–face recognition process

The binary testing process has been designed in order to detect two different states: the subject t is authenticated by the person j in the available database or t is not authenticated by j. These two states are formally described by the statistical hypothesis indicated with $\mathcal{H}_0$ and $\mathcal{H}_1$:

$$\begin{aligned} \mathcal{H}_0 &: t = j \\ \mathcal{H}_1 &: t \neq j. \end{aligned} \quad (4)$$

Statistical model of data given by (1)–(2), indicates that available coordinates are affected by measurement uncertainty σ^2, and thus available data randomly differ from its corresponding expected value. When $\mathcal{H}_0$ holds true, unknown expected values of measured data overlap with Cartesian coordinates of the gallery–face j, i.e. $\mu_{x_{i_t}} = \mu_{x_{i_j}}$, $\mu_{y_{i_t}} = \mu_{y_{i_j}}$, for $i = 1, ..., N_p$. On the other hand, when $\mathcal{H}_1$ holds, thus there is at least a measured landmark for which the corresponding true value is different from the homologous gallery–face coordinate.

It follows that the adopted statistical model uses expected values of available measurements for the authentication process. The set of unknown parameters for the person t is indicated by:

$$\boldsymbol{\theta}_t = [\mu_{x_{1t}}, \mu_{y_{1t}}, \mu_{x_{2t}}, \mu_{y_{2t}}, ..., \mu_{y_{N_P t}}]^T \quad (5)$$

and the hypothesis test that models the authentication issue between t and j can be expressed equivalently in the parametric form as:

$$\begin{aligned} \mathcal{H}_0 &: \boldsymbol{\theta}_t = \boldsymbol{\theta}_j \\ \mathcal{H}_1 &: \boldsymbol{\theta}_t \neq \boldsymbol{\theta}_j. \end{aligned} \quad (6)$$

It is worth to recall that $\boldsymbol{\theta}_t$ is unknown while $\boldsymbol{\theta}_j$ is known.

This authentication problem is addressed in this paper by using the LRT–based technique that follows the NP approach, a method based on the comparison of the data log–likelihood ratio (LLR) with a given threshold value, γ, and that guarantees *a–priori* optimal recognition performance in terms of PTR and PFR [8]. In particular, the optimality is given with respect to PFR since LRT assures the minimum theoretical achievable PFR value for a target (and given) PTR value of the test.

The LLR of data is defined as the logarithm of the ratio between likelihood functions of data that satisfies the hypothesis $\mathcal{H}_1$ and $\mathcal{H}_0$, respectively. By indicating with $p_{\mathbf{M_t}}(\mathbf{M_t}; \boldsymbol{\theta}_{t0}, \mathcal{H}_0)$ and $p_{\mathbf{M_t}}(\mathbf{M_t}; \boldsymbol{\theta}_{t1}, \mathcal{H}_1)$ the likelihood functions of measured data under each hypothesis, thus the LLR is given by:

$$LLR_j(\mathbf{M_t}) \triangleq 2 \cdot \log\left(\frac{p_{\mathbf{M_t}}(\mathbf{M_t}; \boldsymbol{\theta}_{t1}, \mathcal{H}_1)}{p_{\mathbf{M_t}}(\mathbf{M_t}; \boldsymbol{\theta}_{t0}, \mathcal{H}_0)}\right). \quad (7)$$

where $\boldsymbol{\theta}_{t0}$ and $\boldsymbol{\theta}_{t1}$ indicate the true values of the unknown parameters when $\mathcal{H}_0$ and $\mathcal{H}_1$ are true, respectively.

By using (3) and by considering that $\boldsymbol{\theta}_{t0} = \boldsymbol{\theta}_j$, equation (7) can be equivalently written as:

$$\begin{aligned} LLR_j(\mathbf{M}_t) = -\frac{1}{\sigma^2} \cdot \Bigg\{ \sum_{i=1}^{N_P} \Big[\left(x_{i_t} - \mu_{x_{i_t}}\right)^2 + \left(y_{i_t} - \mu_{y_{i_t}}\right)^2 \\ - \left(x_{i_t} - \mu_{x_{i_j}}\right)^2 + \left(y_{i_t} - \mu_{y_{i_j}}\right)^2 \Big] \Bigg\}. \end{aligned} \quad (8)$$

In (8) expected values of available measurements of t under $\mathcal{H}_1$ are unknown, thus the equation cannot be used for practical purposes in the implementation of the authentication test. To cope with such an issue, the Generalized LRT (GLRT) test, a sub–optimal technique which substitutes in (7) unknown parameters value with the corresponding Maximum Likelihood Estimates (MLEs), is adopted [8], [15].

When $\mathcal{H}_1$ holds, thus the MLE estimate of the unknown parameter is $\widehat{\boldsymbol{\theta}}_{t1} = \mathbf{M_t}$. By substituting in (7) $\boldsymbol{\theta}_{t1}$ with $\widehat{\boldsymbol{\theta}}_{t1}$, thus the log–GLRT test for hypothesis test (6) is:

$$\begin{aligned} GLLR_j(\mathbf{M_t}) &\triangleq 2 \cdot \log\left(\frac{p_{\mathbf{M_t}}\left(\mathbf{M_t}; \widehat{\boldsymbol{\theta}}_{t1}\mathcal{H}_1\right)}{p_{\mathbf{M_t}}(\mathbf{M_t}; \boldsymbol{\theta}_{t0}, \mathcal{H}_0)}\right) \\ &= \sum_{i=1}^{N_P}\left[\frac{\left(x_{i_t} - \mu_{x_{i_j}}\right)^2}{\sigma^2} + \frac{\left(y_{i_t} - \mu_{y_{i_j}}\right)^2}{\sigma^2}\right]. \end{aligned} \quad (9)$$

Equation (9) uses available probe–data and known gallery–face coordinates and thus can be implemented in a digital signal processing device.

The GLRT test decides that $\mathcal{H}_0$ is true, i.e. authenticates t with j, if the condition

$$GLLR_j(\mathbf{M_t}) < \gamma \quad (10)$$

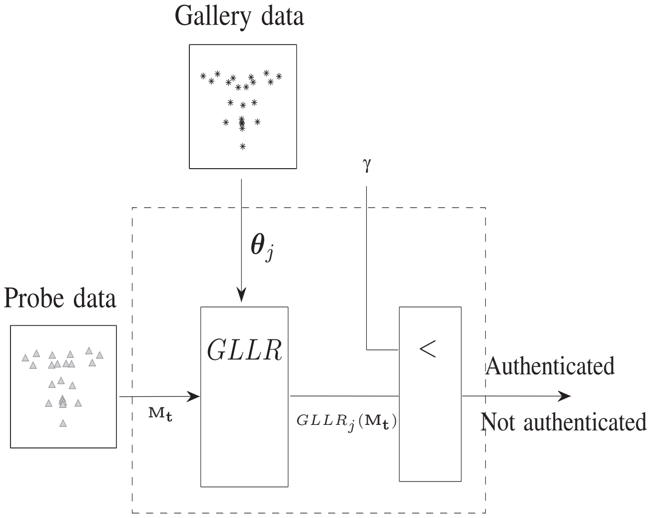

Fig. 2. Block–scheme of the GLRT test for authenticating probe data with gallery–data. Measured data $\mathbf{M_t}$ are processed together to available gallery data $\boldsymbol{\theta}_j$ by means of (9). The outcome is compared to the threshold γ. Value of γ is determined by setting the desired PTR in (15).

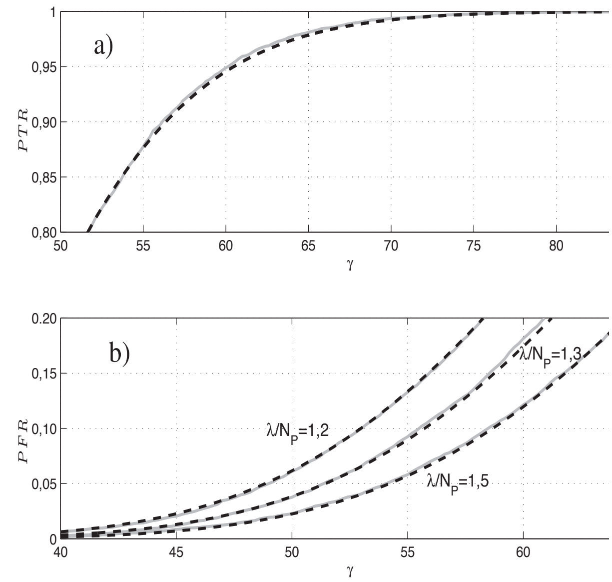

Fig. 3. Behavior of PTR (a) and of PFR (b) of the test (10) versus γ for $N_P = 22$. The three lines in (b) correspond to different value of λ_j/N_P as indicated by the corresponding labels. Black–dashed lines refer to the theoretical curves (15) and (16), grey–bolded lines represent estimates of PTR and PFR by means of Monte–Carlo simulations when σ^2=$2 \cdot 10^{-4}$.

holds true for a given γ value, otherwise t is assumed to be different from j [8]. The block scheme of the test procedure is shown in Fig.2: probe and gallery features are processed by (9) and if the corresponding outcome lower than γ thus t is authenticated by j.

In this context, the probability of correctly recognizing t is given by the probability that (10) holds true when identities of t and j are effectively the same. On the other hand, the test PFR is the probability that (10) holds true when identities of t and j are claimed to be different. Formally:

$$PTR = Pr\{GLLR_j(\mathbf{M_t}) < \gamma; \mathcal{H}_0\} \quad (11)$$
$$PFR = Pr\{GLLR_j(\mathbf{M_t}) < \gamma; \mathcal{H}_1\}. \quad (12)$$

The γ value to be used in the test procedure depends on the target PTR of the test and the relationship between γ and PTR can be deduced by analyzing statistical properties of the detector (9) [8]. Next section describes a detailed analysis for the choice of test parameters that *a–priori* guarantee the target PTR and PFR values.

IV. Theoretical ROC of GLRT for authentication from 2D–landmarks

Statistical properties of the GLRT–based test (10) are derived by analyzing (9) and the statical model of measured data (1)–(2). Data are Normal random variables, thus $GLLR_j(\mathbf{M_t})$ presents a Chi–squared probability density function with $[2 \cdot N_P]$ degrees of freedom and non–central parameter λ_{tj} [8]:

$$GLLR_j(\mathbf{M_t}) \sim \chi^2_{[2 \cdot N_P]}(\lambda_{tj}), \quad (13)$$

$$\lambda_{tj} = \sum_{i=1}^{N_P} \left(\frac{(\mu_{x_{i_t}} - \mu_{x_{i_j}})^2}{\sigma^2} + \frac{(\mu_{y_{i_t}} - \mu_{y_{i_j}})^2}{\sigma^2} \right). \quad (14)$$

The parameter λ_{tj} represents the degree of likeness between t and j when the position of N_P landmarks of t are measured with uncertainty value equal to σ^2. In fact, if face–features of t and j are similar, thus true values of corresponding measurements of t, $\mu_{x_{i_t}}$ and $\mu_{y_{i_t}}$, are close to the homologous gallery–face coordinates, $\mu_{x_{i_j}}$ and $\mu_{y_{i_j}}$, and thus λ_{tj} is small.

If $\mathcal{H}_0$ is true, thus $\lambda_{tj} = 0$ and the statistic of the detector (9) is $GLLR_j(\mathbf{M_t}) \sim \chi^2_{[2 \cdot N_P]}(0)$. In this case, the test correctly recognizes that $t = j$ if (10) holds true and thus the recognition probability can be evaluated as it follows:

$$PTR = Pr\{t = j; \mathcal{H}_0\} = Pr\{GLLR_j(\mathbf{M_t}) < \gamma; \mathcal{H}_0\}$$
$$= F_{\left\{\chi^2_{[2 \cdot N_P]}(0)\right\}}(\gamma) \quad (15)$$

where $F_{\{p(\cdot)\}}(\cdot)$ is the left–tail probability function of the random variable with probability density function $p(\cdot)$.

Eq. (15) shows that the PTR value of the GLRT test (10) depends on the number of available measurements, N_P, and on the threshold value, γ. This relationship can be used for choosing the threshold that guarantees a target PTR value for a given N_P measurement data. As an example, the $N_P = 22$ landmarks in the Bosphorous database are considered. The corresponding behavior of PTR versus γ is shown in Fig.3(a) with a black–dashed line for PTR values ranging from 80% to 100%. This figure shows that PTR increases with γ and that the minimum threshold value to be used in order to guarantee a PTR at least equal to 80% is $\gamma_{min} = 51,7$.

It follows that test (10) applied to to data that can be described by (1)–(2) with $\sigma^2 = 2 \cdot 10^{-4}$ when $\gamma = 51,7$ guarantees a PTR at least equal to 80%.

If the hypothesis $\mathcal{H}_1$ is true, i.e. expected values of measurement data are not equal to coordinates of the j–th person of the database, thus $\lambda_{tj} \neq 0$ and $GLLR_j(\mathbf{M_t}) \sim \chi^2_{[2 \cdot N_P]}(\lambda_{tj})$. In this case if test (10) holds true, thus the test wrongly recognizes t as the the person j, and the corresponding PFR

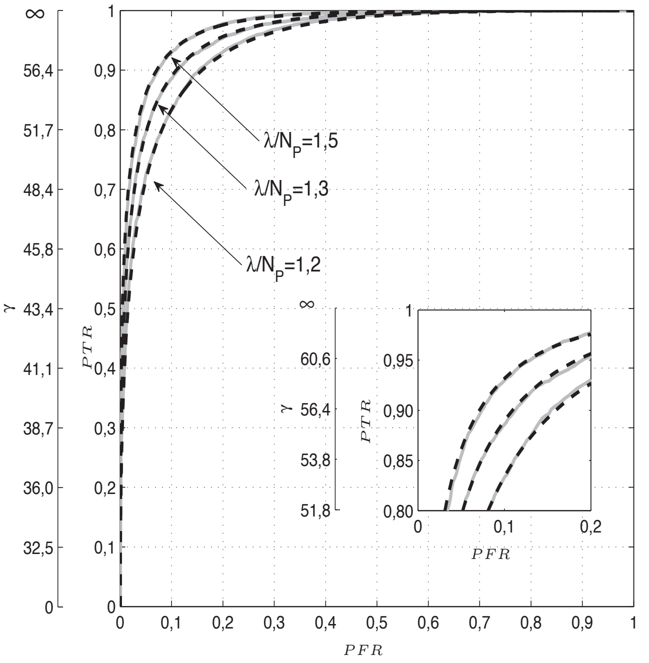

Fig. 4. Behavior of three Receiving Operation Characteristics (ROCs) of the test (10) for different values of the normalized likeness parameters when $N_P = 22$ and σ^2=$2 \cdot 10^{-4}$. Black dashed lines represent theoretical behavior evaluated by means of (15) and (16), gray lines represents results of Monte–Carlo simulations on 10^4 pairs of data. The ordinate axis has been labeled also by γ values corresponding to PTR as given by (15).

value is the probability that $GLLR_j(\mathbf{M_t}) < \gamma$ when $\mathcal{H}_1$ is verified. Formally:

$$\begin{aligned} PFR &= Pr\{t = j; \mathcal{H}_1\} = Pr\{GLLR_j(\mathbf{M_t}) < \gamma; \mathcal{H}_1\} \\ &= F_{\left\{\chi^2_{[2 \cdot N_P]}(\lambda_{tj})\right\}}(\gamma). \qquad (16) \end{aligned}$$

Eq. (16) shows that the PFR of the test (10) depends on values of N_P, γ and λ_{tj}.

Black–dashed lines in Fig.3(b) show the behavior of PFR versus γ when $N_P = 22$ for three different values of the likeness parameter normalized to the used N_P, $\overline{\lambda}_{tj} \triangleq \lambda_{tj}/N_P$, as indicated by the corresponding labels, for PFR ranging from 0 to 20%. This figure shows that also PFR, like PTR, increases with γ, although, for a chosen γ value – i.e. for a given PTR value – the corresponding PFR can be reduced by relaxing the constraint on the normalized target likeness parameter, $\overline{\lambda}_{tj}$, to be discriminated.

As an example, by considering $N_P = 22$, $\gamma = 51,7$, i.e. the value that guarantees $PTR \geq 80\%$ when $N_P = 22$, thus the PFR of the test depends on $\overline{\lambda}_{tj}$; in particular, when $\overline{\lambda}_{tj} = 1,2$ (i.e. $\lambda_{tj} = 26,1$) thus $PFR = 7\%$, when $\overline{\lambda}_{tj} = 1,5$ (i.e. $\lambda_{tj} = 33,1$) thus $PFR = 3\%$.

In order to validate theoretical test performance indicated by (15)–(16), a Monte Carlo approach has been designed on two sets of $N_R = 10^4$ simulated data, the first follows $\mathcal{H}_0$, the second obeys $\mathcal{H}_1$. Estimates of the corresponding PTR and PFR, for many values of γ, have been obtained by counting the number of true and false recognitions, respectively, and by normalizing obtained values to the used N_R [8]. The behavior of simulated PTR and PFR versus γ is also shown in Fig.3 with gray–bolded lines. The good agreement between simulated and theoretical curves confirms the validity of the presented statistical performance evaluation.

The theoretical (black) and simulated (gray) PTR and PFR curves in Fig.3, valid when $N_P = 22$, have been rearranged together as shown Fig.4 thus obtaining the Receiving Operating Characteristics (ROC) of the GLRT test (10) which is the behavior of PTR versus PFR. In particular, the three lines correspond to different normalized likeness parameter as indicated by labels. The ordinate of this figure has been labeled by both PTR and its corresponding γ value. Specifically, the γ value corresponding to each PTR has been obtained by applying (15) with $N_P = 22$.

Information included in the ROC curves can be used for designing the authentication procedure that guarantees the minimum PFR value for a given PTR

As an example, $PTR = 95\%$ is guaranteed by setting $\gamma = 60,6$. By inspecting Fig.4 it can be deduced that the corresponding PFR is determined by the likeness degree desired for the recognition test. In fact, if $\gamma = 60,6$ thus $PFR = 17\%$ is guaranteed when $\overline{\lambda}_{tj} = 1,3$. Moreover, if the target likeness parameter is relaxed to $\overline{\lambda}_{tj} = 1,5$, thus the PFR of the test decreases to 12% while the PTR value doesn't change.

V. An application example to Bosphorous database

The GLRT test has been applied successfully to the Bosphorous database normalized as described in sec.II and some of the test outcomes are reported in Tab.I for the probe–test t and the face–gallery j indicated in each line. In particular, the first four results regard the authentication of the same person with neutral expression by using landmarks extracted from two different images, thus data satisfies $\mathcal{H}_0$. The last four outcomes have been obtained by using landmarks of persons with claimed different identity and thus $\mathcal{H}_1$ holds. The corresponding $GLLR_j(\mathbf{M_t})$ have been evaluated by using (9) with $\sigma^2 = 2 \cdot 10^{-4}$.

The threshold $\gamma = 60,6$ authenticates correctly the subjects 087_N_N_0 (n. 1), 055_N_N_0 (n. 2), 093_N_N_0

TABLE I
$GLLR_j$ EXPERIMENTAL OUTCOMES FOR THE PROBE–TEST t AND THE GALLERY–FACE j IN THE BOSPHOROUS DATABASE.

	t	j	$GLLR_j$	Test outcome if γ=60,6 i.e. PTR=95%
1	087_N_N_0	087_N_N_3	5,10	Authenticated
2	055_N_N_0	055_N_N_2	15,28	Authenticated
3	067_N_N_0	067_N_N_3	95,93	Rejected
4	093_N_N_0	093_N_N_2	46,24	Authenticated
5	033_N_N_0	088_N_N_0	30,83	Authenticated
6	055_N_N_0	077_N_N_0	93,79	Rejected
7	061_N_N_0	011_N_N_0	218,83	Rejected
8	010_N_N_0	029_N_N_0	54,03	Authenticated

(n. 4) and wrongly reject subject 067_N_N_0 (n. 3). On the other hand, the same threshold value wrongly validates probes 033_N_N_0 against 088_N_N_0 (n. 5) and 010_N_N_0 against 029_N_N_0 (n. 8) while it correctly rejects 061_N_N_0 against 011_N_N_0 (n. 7) and 055_N_N_0 versus 077_N_N_0 (n. 6).

By following the criterion given in sec.V, the used threshold value guarantees $PTR = 95\%$ for all the test outcomes in Tab.I. In order to determine the PFR of each test outcome, the target λ_{tj} must be indicated. Fig.3(b) shows that when $\lambda_{tj} = 1, 5$, i.e. $\lambda_{tj} = 33, 1$, $\gamma = 60, 6$ guarantees a $PFR \leq 12\%$, thus outcomes 1, 2, and 5, smaller than 33, 1, have a $PFR \leq 12\%$ while 4, and 8 presents a larger PFR value. If the minimum λ_{tj} value to be discriminated is reduced thus the PFR of the test increases if the same threshold value is used.

VI. Conclusions

The topic of this paper regards the statistical characteriza-tion of an LRT–based algorithm that processes few landmarks coordinates anyway extracted from face–images with neutral expression for recognizing if a human–face belongs to a given database. The proposed algorithm follows the LRT approach and evaluates the sum of squared distance between homol-ogous measurements and given gallery coordinates weighted by the measurement uncertainty value. The authentication is performed on the basis of a threshold criterion. Although this featured–based algorithm is known and already used, however scientific literature sets the comparison threshold value by following an empirical criterion on the basis of the used database or on the feature–extraction method [1], [2].

In this paper, the LRT–based algorithm has been characterized in terms of P T R and P F R and the corresponding theoretical expressions, which have been validated by means of Mote–Carlo simulations, have been given in a closed form and can be applied to any database. Theoretical curves are parametrized on the number of used landmarks, the used threshold value and the likeness degree between subjects to be discriminated. These parameters have been used for defining *a–priori* a criterion for choosing the threshold value, γ, that assures a given P T R and a likeness degree corresponding to a target P F R. Some application examples that use data of the Bosphorous database have been described and reliability of obtained results has been discussed.

Since statistical performance of the proposed authentication algorithm are guaranteed independently of the method used for extracting features, thus the designed LRT–based technique can be used also for comparing effectiveness of different features extraction algorithms.

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